# Homework 5

cs249
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Due:

## 5.1 Gravity

1. Type in the Euler example from the book (the one that finds the position of a falling object). Put it in a file called `gravity.m` and change the name of the function accordingly. Test it and compare the results to the results in the book.

2. Run the code with smaller and smaller time steps until the result agrees with the exact answer to three significant digits. Roughly what is the largest time step that yields three digits of accuracy?

3. On the moon the acceleration of gravity is roughly 1/6 of what it is on Earth. Change the parameter `g` to 1.6 and run the program again. Compute the analytic result $y = 1/2gt^2$ for the new value of g and $t = 2$. Again, how small does the time step have to be to yield three digits of accuracy. Does the parameter `g` affect the accuracy of the algorithm?

4. Modify the program so that as it runs it accumulates the values of `t` in an array named `ts` and the values of `y` in an array named `ys`. At the end of the program, plot `ys` versus `ts`. Print a copy of the plot. Does it have the shape you expected?

5. Modify the program so that instead of using the old value of `t` to compute the velocity, it uses the new value. What effect does the change have on the result? How does the change affect the accuracy of the result?

6. Modify the program one more time so that instead of using the new value of `t` it uses the midpoint between the old and new values. How does the change affect the accuracy of the result?

## 5.2 Coffee

1. Read the handout from Gould and Tobochnik about the coffee cooling problem.

2. Write a MATLAB program to compute the temperature of the coffee as a function of time, given the starting temperature. The cooling constant, $r$, should be a parameter of the program. You should make it easy to adjust the time step and the total time of the simulation.

3. Set the initial temperature to 82.3 degrees Celcius and the total time to 46 seconds. Choose a value of $r$ and run the program. Compare your results to the table on page 21 of Gould and Tobochnik. Adjust the value of $r$ to match the value in the table as closely as possible.

4. Set the initial temperature to 68.8 degrees and run the program again. Compare the results to the second column in the table. Can you find a value of $r$ that matches both experimental results fairly closely? If not, can you think of a reason $r$ might be different for white and black coffee? If there are discrepancies between the output of your model and the experimental results, what do you think the causes are?

5. Finally, we are ready to answer the question. Imagine that the initial temperature is 90 degrees and that you don't want to drink the coffee until it is 75 degrees. Assume that adding cream drops the temperature of the coffee by 5 degrees, regardless of when it is added.

   If we add the cream immediately, the initial temperature is 85 degrees. Compute how long it takes to reach 75 degrees. Try to find an answer that is accurate to 2 digits.

   If we wait to add the cream, the initial temperature is 90 degrees. Compute how long it takes the coffee to reach 80 degrees. Again, try to get 2 digits of accuracy.

   Which method is faster? Is the result consistent with your expectation?

6. Are there any simplifying assumptions in the model that might affect the result? For example, what if the estimates of $r$ are inaccurate. How much margin for error is there before the inaccuracy affects the qualitative result? Are there any other sources of error that you think might be significant?