

Name: _____

Exam 3

Foundations of Computer Science
Fall 2007

This exam is open book, open notes, open computer. You may use the Prolog interpreter to check your answer to Part 2, but don't spend too much time on it.

You may use your laptop to access general documentation, but **not to search for answers to questions on this exam.**

1 Decidability (12 points)

1. Let $A = \{ \langle P, Q \rangle \mid P \text{ and } Q \text{ are PDAs and } L(P) \subset L(Q) \}$. Sketch a proof that A is not decidable.

2. Let $PD = \{ \langle D, w \rangle \mid D \text{ is a DFA, } w \text{ is a string in } L(D) \text{ and } w \text{ can be pumped.} \}$. Show that PD is decidable.

2 Prolog (12 points)

One way to represent a graph in Prolog is with a fact for each edge:

```
edge(1,2).  
edge(1,4).  
edge(1,3).  
edge(2,3).  
edge(2,5).  
edge(3,4).  
edge(3,5).  
edge(4,5).
```

To handle undirected graphs, you can add a rule that says that two vertices are connected if there is an edge between them in either direction:

```
connected(X,Y) :- edge(X,Y) ; edge(Y,X).
```

1. Draw a diagram of the undirected graph represented by the rules above.

2. Write a set of rules for a predicate, `clique`, that takes a list of vertices and checks whether the vertices form a clique. For example, in the graph above, `clique([1,2,3])` is `true` and `clique([1,3,5])` is `no`.

3. Sketch a proof that $ss0 + sss0 = sssss0$ is a theorem of PM.

4 Complexity (12 points)

1. What are the two techniques we have seen for showing that a language is NP-complete?

PLEASE DO ONE OF THE FOLLOWING TWO PROBLEMS. If you do both, indicate clearly which one you want me to grade.

2. Let $\text{DOUBLE-SAT} = \{\langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments}\}$. Show that DOUBLE-SAT is NP-complete.

3. Show that if $P = NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.