COLBY COLLEGE
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Fall 1998

Assignment 4: Initial-value problems

Reading: Faires and Burden Chapter 5.

Due: Thursday 8 October

The purpose of this assignment is to implement some of the techniques for solving IVPs and to continue to expand our use of Maple.

Taylor methods

1. Pick up the Maple worksheet named `hw4_help.mws` from the class web page and read through it. I demonstrates how to use Maple arrays; you should also read the section about arrays in the Maple Learning Guide in the lab. It also defines a new Maple procedure named `plotarray` that is useful for displaying the results of a numerical IVP solution. In future assignments you will be writing Maple procedures, so you might want to note in passing what a procedure definition looks like.

2. Make a copy of this worksheet before you modify it so that you can revert to a known state if things get funny.

3. Modify the worksheet to halve the step size (set $n := 40$)and see what the effect is on the accuracy of the answer.

4. Modify the worksheet to solve Exercise 3d on page 188. To check the accuracy of your answer, use the analytic solution in exercise 4d. This exercise is for your own benefit; you do not need to turn in the result.

5. Starting again with a clean copy of the worksheet, modify it so that it uses the second order Taylor method. To do this you will need a functional operator that evaluates $f'(t, y(t))$ (see page 184). The Functional Operator Handout explains how to do that; it is also available from the web page (it's called `functional_demo.mws`). Use a step size of 0.1 ($n := 20$).

6. When your implementation works, take a minute to clean up the worksheet: delete unnecessary section, suppress the output of trivial operations (by replacing ; with :), and add text sections that explain what you have done. Alternatively, feel free to write things on the printout.

7. Do Exercise 10 on page 189 and plot the results. Hint: the answer should look like a damped oscillation.

Runge-Kutta

1. Starting again from the clean worksheet, implement the 4th Order Runge-Kutta method to solve Exercise 3b on page 188, using a step size of 0.1 ($n := 20$).

2. Compare the errors with the two other methods (Euler and 2nd order Taylor). Make sure you are comparing errors with the same number of steps.

Adams-Bashforth

1. Solve the same problem using Adams-Bashforth Four-Step Explicit Method (page 200). In order to get started, you will have to use 4th order Runge-Kutta to generate the first 4 values.

2. Compare the errors with the other three methods.

Adams-Moulton

1. Solve the same problem using Adams-Moulton Three-Step Implicit Method *as a corrector*. In other words, do not solve the implicit equation at each step. Rather, use Adams-Bashforth Four-Step Explicit Method to generate an estimated solution at each step and then calculate a "corrected" estimate by plugging the "prediction" into the implicit equation.

2. Compare the errors with the other four methods. For a given number of steps, how many times does each method evaluate the function $f'(t, y)$?